Cavity QED: Quantum Control with Single Atoms and Single Photons

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• Quantum networks
• Cavity QED
  - Strong coupling cavity QED
  - Network operations enabled by cavity QED
• Microtoroidal resonators and cold atoms
  - Cavity QED with microtoroids
  - Observation of strong coupling
  - The "bad cavity" regime
  - A photon turnstile dynamically regulated by one atom
  - Future possibilities
Quantum Networks

Quantum node: generation, processing, & storage of quantum information (states)

Matter, e.g., atoms (quantum information stored in internal, electronic states)

Light, e.g., single photons (quantum information stored in photon number or polarisation states)

Quantum channel: transfer & distribution of quantum entanglement

Matter-light interface

Require deterministic, reversible quantum state transfer between material system and light field

Cavity Quantum Electrodynamics (Cavity QED)

\[
H = \omega_{\text{cav}} a^+ a + \omega_{\text{atom}} \sigma^+ \sigma^- + g(a^+ \sigma^- + \sigma^+ a)
\]

**Atom-cavity interaction Hamiltonian**

- **2-level atom**
  - \( \sigma^+ \)
  - \( \sigma^- \)
  - \( |0\rangle \)
  - \( |1\rangle \)

- **Atom-cavity interaction**
  - \( g \approx \mu_{01} E \)
  - \( \mu_{01} \) - atomic transition dipole moment
  - \( E \) - electric field per photon

- **Electric field per photon**
  - \( E \approx \sqrt{\hbar \omega_{\text{cav}} / V_{\text{mode}}} \)

- **Cavity photon number**
  - |0,0⟩

- **Atomic state**
  - |0,1⟩
  - |1,0⟩
  - |1,1⟩
  - |2,0⟩
Strong Coupling Cavity QED

Strong dipole transition in optical cavity of small mode volume, high finesse

\[ g >> \kappa, \gamma \]

Coherent dynamics dominant over dissipative processes

- Nonlinear optics with single photons
- Strong single-atom effects on cavity response
- Controllable manipulation of quantum states

\( \gamma \) - atomic spontaneous emission rate
\( \kappa \) - cavity field decay rate
Network Operations Enabled by Cavity QED

(i) Quantum State Transfer: Atom ↔ Field

(ii) Quantum State Transfer: Node ↔ Node

(iii) Conditional Quantum Dynamics
(i) Quantum State Transfer: Atom ↔ Field


Recent experiments

(ii) Quantum State Transfer: Node ↔ Node


\[
(\alpha \ket{0} + \beta \ket{1})_{\text{atom } 1} \otimes \ket{0}_{\text{atom } 2} \rightarrow \ket{0}_{\text{atom } 1} \otimes (\alpha \ket{0} + \beta \ket{1})_{\text{atom } 2}
\]
(iii) Conditional Quantum Dynamics

Cavity QED with cold neutral atoms (Fabry-Perot resonators)

- H.J. Kimble (Caltech)
- G. Rempe (MPQ, Garching)
- M. Chapman (Georgia Tech)
- D. Stamper-Kurn (Berkeley)
- D. Meschede (Bonn)
- L. Orozco (Maryland)
- ...

Typically

\[
\begin{align*}
g/2\pi & \sim \text{few } \times 10 \text{ MHz} \\
\kappa/2\pi & \sim \text{few MHz} \quad (Q \sim 10^5)
\end{align*}
\]

Cavity QED with trapped ions

- R. Blatt (Innsbruck)
- W. Lange (Sussex)
- C. Monroe (Maryland)
- M. Chapman (Georgia Tech)
- ...

Experimental Cavity QED With Cold Atoms
New Architectures: Optical Microcavities


- Lithographically fabricated
- Integrable with atom chips, scalable networks
Microtoroidal Resonators

Outline:

- Microtoroidal resonators and fiber tapers
  - critical coupling
- Microtoroidal resonators and cold atoms
  - physical setup, basic parameters
  - strong coupling cavity QED
- Experimental observation of strong coupling

- The “bad cavity” regime
- A photon turnstile dynamically regulated by one atom
- Further possibilities
  - single photon transistor
Microtoroidal Resonators + Fiber Tapers


- Coupling through evanescent fields
- 99.97% fiber-taper to microtoroid coupling efficiency!
- Readily integrated into quantum networks
- Ultrahigh Q-factors and small mode volumes
Projected Cavity QED Parameters


Microtoroid of major diameter 10–20 microns:

\[
\begin{align*}
g/2\pi & \sim \text{few } \times 100 \text{ MHz} \\
\kappa_{i}/2\pi & < 1 \text{ MHz } (Q \sim 10^{8-9})
\end{align*}
\]

near surface of toroid
Microtoroidal Resonator - Critical Coupling

Critical coupling condition

\[ \kappa_{\text{ex}} = \kappa_{\text{ex}}^{\text{cr}} = \sqrt{\kappa_i^2 + h^2} \]

\[ \Rightarrow \quad T_F(\Delta_C = 0) = 0 \]

(destructive interference in forward direction)

Output fields

\[ a_{\text{out}} = a_{\text{in}} + \sqrt{2\kappa_{\text{ex}}} \ a \]

\[ b_{\text{out}} = b_{\text{in}} + \sqrt{2\kappa_{\text{ex}}} \ b \]

\[ T_F = \frac{\langle a_{\text{out}}^+ a_{\text{out}} \rangle}{\langle a_{\text{in}}^+ a_{\text{in}} \rangle} \]
• Atoms couple to evanescent field of whispering gallery modes, “disrupt” critical coupling condition
Microtoroid Cavity QED - Basic Parameters

\[ H = \Delta_A \sigma^+ \sigma^- + \Delta_C \left( a^+ a + b^+ b \right) \]
\[ + h \left( a^+ b + b^+ a \right) \left( E_p^* a + E_p a^* \right) \]
\[ + \left( g_{tw}^* a^+ \sigma^- + g_{tw} \sigma^+ a \right) + \left( g_{tw}^* b^+ \sigma^- + g_{tw} \sigma^+ b \right) \]

\( \Delta_A = \omega_A - \omega_p, \quad \Delta_C = \omega_C - \omega_p \)

- Mode-mode coupling \( h \)
- Atom-field coupling

\[ g_{tw}(r,x) = g_0^{tw}(r)e^{ikx} \]
\[ g_0^{tw}(r) \sim e^{-kr} \]

Probe field driving, frequency \( \omega_p \)
Define normal mode operators:  

\[ A = \frac{1}{\sqrt{2}}(a + b), \quad B = \frac{1}{\sqrt{2}}(a - b) \]

\[
H = \Delta_A \sigma^+ \sigma^- + (\Delta_C + h)A^+ A + (\Delta_C - h)B^+ B
+ \frac{1}{\sqrt{2}} \left[ E_p^*(A + B) + E_p(A^+ + B^+) \right]
+ g_A \left( A^+ \sigma^- + \sigma^+ A \right) - ig_B \left( B^+ \sigma^- - \sigma^+ B \right)
\]

\[ g_A = g_0 \cos(kx) \quad g_0 = \sqrt{2} g_0^{tw} \quad g_B = g_0 \sin(kx) \]

Normal modes ↔ **standing waves** around circumference of toroid
Microtoroid Cavity QED

Level structure (vacuum Rabi splitting)

Forward transmission

\[ T_F = \frac{\langle a_{\text{out}}^+ a_{\text{out}} \rangle}{\langle a_{\text{in}}^+ a_{\text{in}} \rangle} \]

Atom-cavity detuning \( \Delta_{AC} \)

- Empty cavity resonance
- Dressed state #1 (Cavity-associated)
- Atom
- Dressed state #2 (Atom-associated)
- Dressed state #3 (Uncoupled optical mode)

 Probe field detuning

- \( kx = 0 \)
- \( kx = \pi/4 \)
- \( kx = \pi/2 \)

 no atom

(\( \Delta_{AC} = 0 \))
Microtoroid Cavity QED

Can use dependence of $T_F$ on $\Delta_{AC}$ to determine $g_0$
Observation of Strong Coupling

\[ g_0^{\max} \approx 2\pi \cdot 50 \text{ MHz} > \begin{cases} 
\kappa_{\text{tot}} \approx 2\pi \cdot 18 \text{ MHz} \\
\gamma_{\perp} = 2\pi \cdot 2.6 \text{ MHz}
\end{cases} \]

Effect of Increasing Cavity Loss

\[ \kappa_{\text{tot}} = \kappa_i + \kappa_{\text{ext}}^{\text{cr}} = \kappa_i + \sqrt{\kappa_i^2 + \hbar^2} \]

- \( \kappa_{\text{tot}} < g_0 \)
- \( \kappa_{\text{tot}} \approx g_0 \)
- \( \kappa_{\text{tot}} \gg g_0 \)

Vacuum Rabi splitting

Cavity-enhanced atomic spontaneous emission
"Bad Cavity" Regime

\[ \kappa_{\text{tot}} \approx 2\pi \cdot 165 \text{ MHz} \gg \begin{cases} g_0^{\text{max}} \approx 2\pi \cdot 70 \text{ MHz} \\ \gamma_\perp = 2\pi \cdot 2.6 \text{ MHz} \end{cases} \]  

(Caltech '07)

- Theory: Adiabatic elimination of cavity modes
- Effective master equation for atomic density matrix:

\[ \dot{\rho}_A = -i[H_A, \rho_A] + \frac{\Gamma}{2} \left( 2\sigma^- \rho_A \sigma^+ - \sigma^+ \sigma^- \rho_A - \rho_A \sigma^+ \sigma^- \right) \]

\[ H_A = \Delta_A \sigma^+ \sigma^- + \left( \Omega_0 \sigma^+ + \Omega_0^* \sigma^- \right) \]

- Cavity-enhanced atomic spontaneous emission rate

\[ \Gamma \approx \gamma + \frac{2g_0^2}{\kappa_{\text{tot}}} = \gamma (1 + 2C), \quad C = \frac{g_0^2}{\kappa_{\text{tot}} \gamma} \]

single-atom "cooperativity" parameter
Output Fields: Bad Cavity Regime

\[
\begin{align*}
    a_{\text{out}} &= a_{\text{in}} + \sqrt{2\kappa_{\text{ex}}} a \quad \rightarrow \quad \alpha_0 + \alpha_\sigma_-
    \\
    b_{\text{out}} &= b_{\text{in}} + \sqrt{2\kappa_{\text{ex}}} b \quad \rightarrow \quad \beta_0 + \beta_\sigma_-
\end{align*}
\]

\[
\begin{bmatrix}
    \alpha_0 \\
    \beta_0
\end{bmatrix} = \text{coherent amplitudes without atom}
\]
Forward/Backward Spectra

Central atomic resonance, width $\approx \Gamma$

$$
\begin{align*}
g_0^{\text{tw}} / 2\pi &= 50 \text{ MHz} \\
(\kappa_i, \kappa_{\text{ext}})/2\pi &= (75, 90) \text{ MHz} \\
h/2\pi &= 50 \text{ MHz}
\end{align*}
$$

Different azimuthal positions $\chi$
A Photon "Turnstile"

Bad cavity regime

\[ a_{\text{out}} \rightarrow \alpha_0 + \alpha_{-}\sigma^- \]
\[ b_{\text{out}} \rightarrow \beta_0 + \beta_{-}\sigma^- \]

- **Critical coupling:** \( \alpha_0(\Delta_C \approx 0) \approx 0, \beta_0(\Delta_C \approx 0) \neq 0 \)
- `1st’ photon transmitted into \( a_{\text{out}} \) can only originate from atom
- Emission projects atom into ground state
- `2nd’ photon cannot be transmitted until atomic state regresses to steady-state, time scale \( 1/\Gamma \)

\[ \Rightarrow \text{excess photons } \text{`rerouted’ to } b_{\text{out}} \]

Microtoroid-atom system only transmits photons in the forward direction one-at-a-time
Note: Other photon turnstile devices


Blockade a structural effect due to anharmonicity of energy spectrum for multiple excitations

Microtoroid-atom system: blockade regulated dynamically by conditional state of one atom
→ efficient mechanism, insensitive to many experimental imperfections
Intensity Correlation Functions

\[ g_F^{(2)} = \frac{\langle a_{\text{out}}^+ a_{\text{out}}^2 \rangle}{\langle a_{\text{out}}^+ a_{\text{out}}^* \rangle^2}, \quad g_B^{(2)} = \frac{\langle b_{\text{out}}^+ b_{\text{out}}^2 \rangle}{\langle b_{\text{out}}^+ b_{\text{out}}^* \rangle^2} \]

(probabilities of “simultaneous” photon detections)

\[ \langle (a_{\text{out}}^+)^2 a_{\text{out}}^2 \rangle \sim \langle \sigma^+ \sigma^- \rangle = 0 \]

antibunching at \( \Delta \approx 0 \)

bunching at \( \Delta \approx 0 \)
• Cross correlation $\xi_{12}(\tau)$
• $\xi_{12}(\tau) > \xi_{12}(0)$ a prima facie observation of nonclassical light
Observation of Antibunching/Turnstile Effect

- Analysis of single and joint detections at $D_{1,2}$ conditioned on single atom transit

$$g_F^{(2)}(\tau) \approx \left(1 - e^{-\Gamma/2}\right)^2, \quad 1/\Gamma \approx 2.8 \text{ ns } (C \sim 5)$$

“Blockade” effect robust, e.g., requires only

$$\frac{2g(\vec{r})^2}{K_{\text{tot}}'} > 1$$

In the Future ...

- Minimise intrinsic losses
  \[ \kappa_i \ll \kappa_{ex} \]
- Large mode-mode coupling \( h \)

\[ \Rightarrow \text{Near-ideal input/output} \]
Microtoroid + Atom: **Over-Coupled** Regime

**Bad cavity regime**

- **Strong over-coupling:** $\kappa_{\text{ex}} \gg h$, $\kappa_{i}$ ($\kappa_{\text{tot}} \approx \kappa_{\text{ex}}$)
- **No atom ($\alpha = \beta = 0$):** strong transmission, small reflection ($\beta_{0} \approx 0$)
- **With atom:** destructive interference between $\alpha_{0}$ and $\alpha_{-\sigma^{-}}$

  $\Rightarrow$ strong reflection, small transmission

\[ a_{\text{out}} \rightarrow \alpha_{0} + \alpha_{-\sigma^{-}} \]
\[ b_{\text{out}} \rightarrow \beta_{0} + \beta_{-\sigma^{-}} \]
Spectra and Correlations: Over-Coupled Regime

Transmission and Reflection

\[ T_B(\Delta_C = 0) \approx \left( \frac{\kappa_{\text{ex}}}{\kappa_{\text{tot}}} \right)^2 \left( \frac{2C}{1 + 2C} \right)^2 \]

\[ K_{\text{tot}} \approx K_{\text{ex}} \]

\[ C \sim \frac{g_0^2}{\kappa_{\text{tot}}^\gamma} \gg 1 \]

Single atom cooperativity

antibunching in reflected field
... and beyond

- Controlled interactions of photons
- Trapping of atoms close to toroid
- Multi-toroid/atom systems
  - Scalable quantum processing on atom chips
Microdisk-Quantum Dot Systems

Cast

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