Collège de France abroad Lectures

Quantum information with real or artificial atoms and photons in cavities

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A series of six lectures at NUS in the Course « QT5201E Special Topics in quantum information: advanced quantum optics »
Goal of lectures

Manipulating states of simple quantum systems has become an important field in quantum optics and in mesoscopic physics, in the context of quantum information science. Various methods for state preparation, reconstruction and control have been recently demonstrated or proposed.

Two-level systems (qubits) and quantum harmonic oscillators play an important role in this physics. The qubits are information carriers and the oscillators act as memories or quantum bus linking the qubits together. Coupling qubits to oscillators is the domain of Cavity Quantum Electrodynamics (CQED) and Circuit Quantum Electrodynamics (Circuit-QED). In microwave CQED, the qubits are Rydberg atoms and the oscillator is a mode of a high Q cavity while in Circuit QED, Josephson junctions act as artificial atoms playing the role of qubits and the oscillator is a mode of an LC radiofrequency resonator.

The goal of these lectures is to analyze various ways to synthesise non-classical states of qubits or quantum oscillators, to reconstruct these states and to protect them against decoherence. Experiments demonstrating these procedures will be described, with examples from both CQED and Circuit-QED physics. These lectures will give us an opportunity to review basic concepts of measurement theory in quantum physics and their links with classical estimation theory.
Outline of lectures

• Lecture 1 (February 6\textsuperscript{th}): Introduction to Cavity QED with Rydberg atoms interacting with microwave fields stored in a high Q superconducting resonator.

• Lecture 2 (February 8\textsuperscript{th}): Review of measurement theory illustrated by the description of quantum non-demolition (QND) photon counting in Cavity QED.

• Lecture 3 (February 10\textsuperscript{th}): Estimation and reconstruction of quantum states in Cavity QED experiments; the cases of Fock and Schrödinger cat stats.

• Lecture 4 (February 13\textsuperscript{th}): Quantum feedback experiments in Cavity QED preparing and protecting against decoherence non-classical states of a radiation field.

• Lecture 5 (February 15\textsuperscript{th}): An introduction to Circuit-QED describing Josephson junctions as qubits and radiofrequency resonators as quantum oscillators.

• Lecture 6 (February 17\textsuperscript{th}): Description of Circuit-QED experiments synthesizing arbitrary states of a field oscillator.
I-A

The basic ingredients of Cavity QED: qubits and oscillators

Two-level system (qubit)  Field mode (harmonic oscillator)
Description of a qubit (or spin 1/2)

Any pure state of a qubit (0/1) is parametrized by two polar angles \( \theta, \varphi \) and is represented by a point on the Bloch sphere:

\[
|\theta, \varphi\rangle = \cos \frac{\theta}{2} e^{-i \varphi/2} |0\rangle + \sin \frac{\theta}{2} e^{i \varphi/2} |1\rangle
\]

A statistical mixture is represented by a density operator:

\[
\rho = \sum_i p_i |\psi_{\text{qubit}}^{(i)}\rangle \langle \psi_{\text{qubit}}^{(i)}| \quad (\text{pure state})
\]

\[
\rho = \sum_i p_i |\psi_{\text{qubit}}^{(i)}\rangle \langle \psi_{\text{qubit}}^{(i)}| \quad \left( \sum_i p_i = 1 \right) \quad (\text{mixture})
\]

which can be expanded on Pauli matrices:

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

\[
\sigma_i^2 = I \quad (i = x, y, z)
\]

\[
\rho = \frac{1}{2}(I + \vec{P} \cdot \vec{\sigma}) \quad ; \quad |\vec{P}| \leq 1
\]

\( \rho \) hermitian, with unit trace and \( \geq 0 \) eigenvalues

\[
|\vec{P}| = 1: \text{pure state} \quad ; \quad |\vec{P}| < 1: \text{mixture} \quad ; \quad |\vec{P}| > 1: \text{non physical (negative eigenvalue)}
\]

A qubit quantum state (pure or mixture) is fully determined by its Bloch vector
Description of a qubit (cont’d)

The Bloch vector components are the expectation values of the Pauli operators:

\[ P_i = \text{Tr} \rho \sigma_i = \langle \sigma_i \rangle \quad (i = x, y, z) \quad ; \quad \rho = \frac{1}{2} \left[ I + \sum_i \langle \sigma_i \rangle \sigma_i \right] \]

\((\text{Tr} \sigma_i \sigma_j = 2\delta_{ij})\)

The qubit state is determined by performing averages on an ensemble of realizations: the concept of quantum state is statistical.

Calling \(p_{i+}\) et \(p_{i-}\) the probabilities to find the qubit in the eigenstates of \(\sigma_i\) with eigenvalues \(\pm 1\), we have also:

\[ P_i = p_{i+} - p_{i-} \quad ; \quad \rho = \frac{1}{2} \left[ I + \sum_i (p_{i+} - p_{i-}) \sigma_i \right] \]

A useful formula: overlap of two qubit states described by their Bloch vectors:

\[ S_{12} = \text{Tr} \{ \rho_1 \rho_2 \} = \frac{1}{2} \left[ 1 + \overrightarrow{P_1} \cdot \overrightarrow{P_2} \right] \]

Manipulation and measurement of qubits: Qubit rotations are realized by applying resonant pulses whose frequency, phase and durations are controlled. In general, it is easy to measure the qubit in its energy basis (\(\sigma_z\) component). To measure an arbitrary component, one starts by performing a rotation which maps its Bloch vector along \(0z\), and then one measures the energy.
Qubit rotation induced by microwave pulse

Coupling of atomic qubit with a classical field:

\[ H = H_{at} + H_{int}(t) \quad ; \quad H_{at} = \hbar \frac{\omega_{eg}}{2} \sigma_z \quad ; \quad H_{int}(t) = -D.E_{mw}(t) \]

Microwave electric field linearly polarized with controlled phase \( \varphi_0 \):  

\[ E_{mw}(t) = E_0 \cos(\omega_{mw} t - \varphi_0) = \frac{E_0}{2} \left[ \exp(i(\omega_{mw} t - \varphi_0)) + \exp(-i(\omega_{mw} t - \varphi_0)) \right] \]

Qubit electric-dipole operator component along field direction is off-diagonal and real in the qubit basis (without loss of generality):

\[ D_{\text{along field}} = d\sigma_x \quad (d \quad \text{real}) \]

Hence the Hamiltonian:

\[ H = \hbar \frac{\omega_{eg}}{2} \sigma_z - \hbar \frac{\Omega_{mw}}{2} \sigma_x \left[ \exp(i(\omega_{mw} t - \varphi_0)) + \exp(-i(\omega_{mw} t - \varphi_0)) \right] \quad ; \quad \Omega_{mw} = d.E_0 / \hbar \]

and in frame rotating at frequency \( \omega_{mw} \) around \( Oz \):

\[
\frac{i\hbar}{dt} \begin{split} \frac{d}{dt} \begin{matrix} \Psi \end{matrix} \end{split} = H \begin{matrix} \Psi \end{matrix} \quad ; \quad \begin{matrix} \tilde{\Psi} \end{matrix} = \exp\left( i \frac{\sigma_z \omega_{mw}}{2} t \right) \begin{matrix} \Psi \end{matrix} \rightarrow \frac{i\hbar}{dt} \begin{matrix} \tilde{\Psi} \end{matrix} = \tilde{H} \begin{matrix} \tilde{\Psi} \end{matrix} \quad ; \quad \\
\tilde{H} = \hbar \frac{\omega_{eg} - \omega_{mw}}{2} \sigma_z - \hbar \frac{\Omega_{mw}}{2} e^{i \frac{\sigma_z \omega_{mw} t}{2}} \sigma_x e^{-i \frac{\sigma_z \omega_{mw} t}{2}} \left[ e^{i(\omega_{mw} t - \varphi_0)} + e^{-i(\omega_{mw} t - \varphi_0)} \right] 
\]
Bloch vector rotation (ctn'd)

\[ \tilde{H} = \hbar \left( \frac{\omega_{eg} - \omega_{mw}}{2} \right) \sigma_z - \hbar \frac{\Omega_{mw}}{2} e^{-i\frac{\sigma_y\omega_{mw}t}{2}} - e^{i\frac{\sigma_y\omega_{mw}t}{2}} \left[ e^{i(\omega_{mw}t - \varphi_0)} + e^{-i(\omega_{mw}t - \varphi_0)} \right] \]

\begin{align*}
\sigma_x e^{-i\frac{\sigma_y\omega_{mw}t}{2}} &= \sigma_x \cos \omega_{mw} t - \sigma_y \sin \omega_{mw} t = \\
&= \left( \frac{\sigma_x + i\sigma_y}{2} \right) e^{i\omega_{mw} t} + \left( \frac{\sigma_x - i\sigma_y}{2} \right) e^{-i\omega_{mw} t} 
\end{align*}

The rotating wave approximation (rwa) neglects terms evolving at frequency ±2ω_{mw}:

\[ \tilde{H}_{rwa} = \hbar \left( \frac{\omega_{eg} - \omega_{mw}}{2} \right) \sigma_z - \hbar \frac{\Omega_{mw}}{2} \left( \sigma_+ e^{iq_0} + \sigma_- e^{-iq_0} \right) ; \quad \sigma_\pm = (\sigma_x \pm i\sigma_y) / 2 \]

The rwa hamiltonian is t-independent. At resonance (ω_{eg} = ω_{mw}), it simplifies as:

\[ \tilde{H}_{rwa} = -\hbar \frac{\Omega_{mw}}{2} \left( \sigma_+ e^{iq_0} + \sigma_- e^{-iq_0} \right) = -\hbar \frac{\Omega_{mw}}{2} \left( \sigma_x \cos \varphi_0 - \sigma_y \sin \varphi_0 \right) \]

A resonant mw pulse of length τ and phase \( \varphi_0 \) rotates Bloch vector by angle \( \Omega_{mw} \tau \) around direction Ou in Bloch sphere equatorial plane making angle \( -\varphi_0 \) with Ox:

\[ \tilde{U}(\tau) = \exp(-i\tilde{H}_{rwa} \tau / \hbar) = \exp(-i\frac{\Omega_{mw} \tau}{2} \sigma_u) \]

\[ \sigma_u = \sigma_x \cos \varphi_0 - \sigma_y \sin \varphi_0 \]

A method to prepare arbitrary pure qubit state from state \( |e> \) or \( |g> \). By applying a convenient pulse prior to detection in qubit basis, one can also detect qubit state along arbitrary direction on Bloch sphere.
Description of harmonic oscillator (phonons or photons)

Particle in a parabolic potential or field mode in a cavity

Phase space (with conjugate coordinates $x, p$ or $E_1, E_2$)

Phase space (with conjugate coordinates $x, p$ or $E_1, E_2$)

Basic formulae with photon annihilation and creation operators

$$[a, a^\dagger] = I ; \quad a|n\rangle = \sqrt{n}|n-1\rangle; \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle ; \quad |n\rangle = \frac{a^\dagger^n}{\sqrt{n!}}|0\rangle$$

$$N = a^\dagger a \quad ; \quad H_{\text{field}} = \hbar \omega N \quad ; \quad e^{iN t} a e^{-iN t} = e^{-i\omega t} a$$

Displacement operator: $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$

Coherent state: $|\alpha\rangle = D(\alpha)|0\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle$

Mechanical or electromagnetic oscillation.

Coupling qubits to oscillators is an important ingredient in quantum information.

Particle in a parabolic potential or field mode in a cavity
Coherent state

Coupling of cavity mode with a small resonant classical antenna located at r=0:

\[ V = -\bar{J}(t)\bar{A}(0) \quad ; \quad J(t) \sim \cos \omega t \quad ; \quad A(0) \sim a + a^\dagger \]

Hence, the hamiltonian for the quantum field mode fed by the classical source:

\[ H_Q = \hbar \omega a^\dagger a + V = \hbar \omega a^\dagger a + \Lambda \left( e^{i\omega t} + e^{-i\omega t} \right) (a + a^\dagger) \]

(\( \Lambda \): constant proportional to current amplitude in antenna)

Interaction representation:

\[ \left| \tilde{\psi}_{field} \right\rangle = \exp(i\omega^\dagger a t) \left| \psi_{field} \right\rangle \rightarrow i\hbar \frac{d\left| \tilde{\psi}_{field} \right\rangle}{dt} = \tilde{H}_Q \left| \tilde{\psi}_{field} \right\rangle \quad \text{with} \quad \tilde{H}_Q = \Lambda \left( e^{i\omega t} + e^{-i\omega t} \right) e^{i\omega a^\dagger at} (a + a^\dagger) e^{-i\omega a^\dagger at} \]

Rotating wave approximation (keep only time independent terms):

\[ \tilde{H}_Q(rwa) = \Lambda (a + a^\dagger) \]

Field evolution in cavity starting from vacuum at t=0:

\[ \left| \tilde{\psi}_{field}(t) \right\rangle = \exp \left( -i \frac{\Lambda}{\hbar} \left[ a + a^\dagger \right] t \right) \left| 0 \right\rangle = \exp \left( \alpha a^\dagger - \alpha^* a \right) \left| 0 \right\rangle = e^{-\alpha a^* / 2} e^{\alpha a^\dagger} e^{-\alpha a} \left| 0 \right\rangle \quad (\alpha = -i\Lambda t / \hbar) \]

We have used Glauber formula to split the exponential of the sum in last expression. Expanding \( \exp(\alpha a^\dagger) \) in power series, we get the field in Fock state basis:

\[ \left| \tilde{\psi}_{field}(t) \right\rangle = e^{-|\alpha|^2 / 2} \sum_n \frac{\alpha^n}{\sqrt{n!}} \left| n \right\rangle \]

Coupling field mode to classical source generates coherent state whose amplitude increases linearly with time.
Coherent state (ctn’d)

Produced by coupling the cavity to a source (classical oscillating current)

\[ |\alpha\rangle = \sum_n C_n |n\rangle, \quad C_n = e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}}, \]

\[ \bar{n} = |\alpha|^2, \quad P(n) = |C_n|^2 = e^{-\bar{n}} \frac{n^{-\bar{n}}}{n!} \]

The larger \( n \), the more classical the field is.

Complex plane representation

Im(\( \alpha \))

Re(\( \alpha \))

Amplitude |\( \alpha \)|

Uncertainty circle (« width » of Wigner function—see next page)

\( \varphi = \varphi_0 - \omega_c t \)
Representations of a field state

Density operator for a field mode:
\[
\rho = \psi_{\text{field}} \langle \psi_{\text{field}} | \text{ (pure state)} \quad \rho = \sum_i p_i \psi_{\text{field}}^{(i)} \langle \psi_{\text{field}}^{(i)} | \quad \left( \sum_i p_i = 1 \right) \text{ (mixture)}
\]

Matrix elements of \( \rho \) can be discrete (\( \rho_{nn} \) in Fock state basis) or continuous (\( \rho_{xx} \) in quadrature basis where \( |x\rangle \) are the eigenstates of \( a + a^\dagger \)). Going from one representation to the other is easy knowing the amplitudes \( \langle x|n\rangle \) expressing the oscillator energy eigenstates in the x basis (Hermite polynomial multiplied by gaussian functions).

Representation in phase space: the Wigner function:
\[
W(x, p) = \frac{1}{\pi} \int du \exp(-2ipu) \langle x + u/2 | \rho | x - u/2 \rangle
\]

\( W \) is a real distribution in \((x,p)\) space (equivalently, in complex plane), whose knowledge is equivalent to that of \( \rho \). The «shadow» of \( W \) on p plane yields the x- distribution in the state.

Ground state

Coherent state

Fock state

\( W<0 : \) non-classical
Coupling a qubit to a quantized field mode: the Jaynes-Cummings Hamiltonian

<table>
<thead>
<tr>
<th>Matter</th>
<th>Radiation</th>
<th></th>
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<tbody>
<tr>
<td>Semi-classical</td>
<td>Classical current</td>
<td>Quantum field mode</td>
</tr>
<tr>
<td>Classical current</td>
<td>Quantum field mode</td>
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<tr>
<td>Quantum (qubit)</td>
<td>Classical field</td>
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</tr>
<tr>
<td>Fully quantum</td>
<td>Quantum (qubit)</td>
<td>Quantum field mode</td>
</tr>
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</table>
Qubit-quantum field mode coupling

Coupling qubit dipole to quantum electric field operator:

\[ V = -D \cdot \vec{E}_Q(0) \quad ; \quad E_Q(0) = iE_0 (a - a^\dagger) \]

\[ V = -i dE_0 \sigma_x (a - a^\dagger) = -i dE_0 (\sigma_+ + \sigma_-) (a - a^\dagger) \]

Rotating wave approximation:

\[ V_{rwa} = -i \frac{\hbar \Omega_0}{2} (\sigma_+ a - \sigma_- a^\dagger) \quad ; \quad \Omega_0 = 2dE_0 / \hbar \]

with:

\[ \sigma_+ = \frac{\sigma_x + i \sigma_y}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = |e\rangle\langle g| \quad ; \quad \sigma_- = \frac{\sigma_x - i \sigma_y}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = |g\rangle\langle e| \]

Vacuum Rabi frequency in cavity mode of volume \( V \):

\[ \langle 0 | E_Q^2(0) | 0 \rangle = E_0^2 \langle 0 | a a^\dagger + a^\dagger a | 0 \rangle = E_0^2 \]

\[ \varepsilon_0 V E_0^2 = \frac{\hbar \omega}{2} \quad \rightarrow \quad E_0 = \sqrt{\frac{\hbar \omega}{2 \varepsilon_0 V}} \]

\[ \Omega_0 = \sqrt{\frac{2d^2 \omega}{\varepsilon_0 \hbar V}} \]

Requires large dipole and small cavity volume \( V \)
Qubit-oscillateur coupling: the Jaynes-Cummings Hamiltonian

\[ H = \hbar \omega_{qb} \frac{\sigma_z}{2} + \hbar \omega_{oh} a^\dagger a - i \frac{\hbar \Omega}{2} \left[ \sigma_+ a - \sigma_- a^\dagger \right] \]

\[ |g,n+1\rangle \rightarrow \Omega \sqrt{n+1} |\pm,n\rangle \]

\[ |g,3\rangle \rightarrow \Omega \sqrt{3} |\pm,2\rangle \]
\[ |g,2\rangle \rightarrow \Omega \sqrt{2} |\pm,1\rangle \]
\[ |g,1\rangle \rightarrow \Omega |\pm,0\rangle \]
\[ |g,0\rangle \rightarrow |g,0\rangle \]

\( \Delta = \omega_{qb} - \omega_{oh} \)

Anticrossing of dressed qubit

\[ E_{\pm,n} = (n+1/2) \hbar \omega_{oh} \pm \frac{\hbar}{2} \sqrt{\Delta^2 + \Omega^2} (n+1) \]

at resonance (\( \Delta = 0 \))

\[ |\pm,n\rangle = \frac{1}{\sqrt{2}} (|e,n\rangle \pm i |g,n+1\rangle) \]
\[ |e,n\rangle \rightarrow \cos \frac{\Omega \sqrt{n+1} t}{2} |e,n\rangle + \sin \frac{\Omega \sqrt{n+1} t}{2} |g,n+1\rangle \]
\[ |g,n+1\rangle \rightarrow -\sin \frac{\Omega \sqrt{n+1} t}{2} |e,n\rangle + \cos \frac{\Omega \sqrt{n+1} t}{2} |g,n+1\rangle \]
Non-Resonant coupling: light shifts in CQED

Anticrossing of dressed qubit

$E_{\pm,n} = \left( n+1/2 \right) \hbar \omega_{oh} \pm \frac{\hbar}{2} \sqrt{\Delta^2 + \Omega^2 (n+1)}$

Second order perturbation theory:

$E_{\pm,n} \approx (n+1/2) \hbar \omega_c \pm \hbar \left( \frac{\Delta}{2} + \frac{\Omega^2 (n+1)}{4\Delta} \right)$

Energy shift due to off-resonant coupling (distance between energy level and asymptot):

$E_{+,n} - E_{e,n} \approx \hbar \frac{\Omega^2 (n+1)}{4\Delta} + ... ;$

$E_{-,n} - E_{g,n+1} \approx - \hbar \frac{\Omega^2 (n+1)}{4\Delta} + ...$

Vacuum shift (Lamb-shift):

$\delta_{g,0} = 0 ; \quad \delta_{e,0} = E_{+0} - E_{e,0} \approx \hbar \frac{\Omega^2}{4\Delta}$

Light shift induced on eg transition by $n$-photons:

$E_{+,n} - E_{-,n-1} = \hbar \omega_{eg} + \hbar \frac{\Omega^2}{2\Delta} n + ... \rightarrow \delta(\omega_{eg}) = \frac{\Omega^2}{2\Delta} n ; \quad \varphi_0 = \frac{\Omega^2 t}{2\Delta}$

$\varphi_0$=Phase shift per photon accumulated during time $t$
Measuring qubit phase shift in CQED: the Ramsey interferometer

Qubit initially in $|e\rangle$

$R_1$ pulse rotates Bloch vector by $\pi/2$ around $Ox$

$R_1$ pulse realizes $\pi/2$ rotation around $Ou$ whose direction depends on $R_2-R_1$ relative phase $\varphi_r$

$R_2$ pulse realizes $\pi/2$ rotation around $Oz$

Qubit phase shift $\varphi_c$ during $C$ crossing amounts to Bloch vector rotation around $Oz$

$\exp\left(-i \frac{\pi}{4} \sigma_x \right)$

$\exp\left(-i \frac{\varphi_c}{2} \sigma_z \right)$

$\exp\left(-i \frac{\pi}{4} \sigma_z \right) = \exp\left(-i \pi \frac{\sigma_x}{4} [\sigma_x \cos \varphi_r - \sigma_y \sin \varphi_r] \right)$
Ramsey interferometer (ctn’d)

A useful formula:
\[ \exp(-i\varphi \sigma_u) = \cos \varphi \ I - i \sin \varphi \ \sigma_u \]  
(for any Pauli operator)

Hence the rotation induced by Ramsey interferometer:

\[
R = \exp\left(-i \frac{\pi}{4} \sigma_u\right) \exp\left(-i \frac{\varphi_c}{2} \sigma_z\right) \exp\left(-i \frac{\pi}{4} \sigma_x\right)
\]

Rotation induced by \( R_2 \)  
Cavity phase-shift  
Rotation induced by \( R_1 \)

\[
= \frac{1}{2} \begin{pmatrix}
1 & -ie^{i\varphi_r} \\
-ie^{-i\varphi_r} & 1
\end{pmatrix}
\begin{pmatrix}
e^{-i\varphi_c/2} & 0 \\
0 & e^{i\varphi_c/2}
\end{pmatrix}
\begin{pmatrix}
1 & -i \\
-i & 1
\end{pmatrix}
\]

\[
= -i \begin{pmatrix}
e^{i\varphi_r/2} \sin\left(\frac{\varphi_c + \varphi_r}{2}\right) & e^{i\varphi_r/2} \cos\left(\frac{\varphi_c + \varphi_r}{2}\right) \\
e^{-i\varphi_r/2} \cos\left(\frac{\varphi_c + \varphi_r}{2}\right) & e^{-i\varphi_r/2} \sin\left(\frac{\varphi_c + \varphi_r}{2}\right)
\end{pmatrix}
\]

and the evolution of \(|e\rangle\) and \(|g\rangle\) states:

\[
R |e\rangle = -ie^{i\varphi_r/2} \sin\left(\frac{\varphi_c + \varphi_r}{2}\right) |e\rangle - ie^{-i\varphi_r/2} \cos\left(\frac{\varphi_c + \varphi_r}{2}\right) |g\rangle
\]

\[
R |g\rangle = -ie^{i\varphi_r/2} \cos\left(\frac{\varphi_c + \varphi_r}{2}\right) |e\rangle - ie^{-i\varphi_r/2} \sin\left(\frac{\varphi_c + \varphi_r}{2}\right) |g\rangle
\]

Ramsey fringes detected by sweeping \( \varphi_r \). Fringe-phase used to measure cavity shift \( \varphi_c \).
General scheme of cavity QED experiments

ENS experiments: Rydberg atoms in states e and g behave as qubits. They are prepared in B in state e and cross one at a time the high-Q cavity C where they are coupled to a field mode. The atom-field system evolution is ruled by the Jaynes-Cummings hamiltonian. A microwave pulse applied in $R_1$ prepares each atom in a superposition of e and g. After C, a second pulse, applied in $R_2$, maps the measurement direction of the qubit along the Oz axis of the Bloch sphere, before detection of the qubit by selective field ionization in an electric field (in D). The $R_1$-$R_2$ combination constitutes a Ramsey interferometer. This set-up has been used to entangle atoms, realize quantum gates, count photons non-destructively, reconstruct non classical states of the field and demonstrate quantum feedback procedure (lectures 1 to 4).
Analogy between Ramsey and Mach Zehnder interferometers

In Ramsey interferometer, two resonant pulses split and recombine atomic state in Hilbert space. Qubit follows two paths between $R_1$ and $R_2$ along which it undergoes different phase-shifts. By finally detecting qubit in $e$ or $g$ and sweeping phase, one get fringes which informs about differential phase shifts between the states.

In Mach-Zehnder, the splitting and recombination occurs on particle trajectories. Beam splitters replace Ramsey pulses. Fringes inform about differential phase shifts induced on the paths.

Detectors in the two outgoing channels detect fringes with opposite phases.
I-C

A special system: circular Rydberg atoms coupled to a superconducting Fabry-Perot cavity
A qubit extremely sensitive to microwaves: the circular Rydberg atom

Atom in ground state:
**electron on 10^{-10} m diameter orbit**

Very long radiative lifetime $T_R$

$$\omega_{n,n-1} \sim n^{-3} ; \quad P_{rad} = \frac{\hbar \omega_{n,n-1}}{T_R} \sim (\omega_{n,n-1} r_R)^2 \sim n^{-8} \rightarrow T_R \sim n^5$$

The localized wave packet revolves around nucleus at the transition frequency (51 GHz) between the two states like a clock's hand on a dial.

The electric dipole is proportional to the qubit Bloch vector in the equatorial plane of the Bloch sphere.

**Rydberg formulae**

$$n\lambda_{\text{AB}} = \frac{nh}{mv} = 2\pi r_R \rightarrow r_R = \frac{nh}{mv} ;$$

$$|E| = \frac{mv^2}{2} = \frac{q^2}{8\pi e_0 r_R} = \frac{q^2}{8\pi e_0 \hbar}$$

$$\rightarrow v = \frac{q^2}{4\pi e_0 \hbar c} n = \alpha_{fs} c = \frac{1}{137} c \rightarrow |E| = \frac{1}{2mc^2 \alpha_{fs}^2}$$

Atom in circular Rydberg state:
**electron on giant orbit**
(tenth of a micron diameter)

Atom in circular Rydberg state: electron on giant orbit
tenth of a micron diameter

**Rydberg states:** $|e\rangle \rightarrow |e\rangle + |g\rangle$

Very long radiative lifetime $T_R$

$$r_R = n^2 \frac{\hbar}{\alpha_{fs} mc} = n^2 a_0$$

**e (n=51)**

**g (n=50)**

Electron is localised on orbit by a microwave pulse preparing superposition of two adjacent Rydberg states: $|e\rangle \rightarrow |e\rangle + |g\rangle$
The path towards circular states: an adiabatic process involving 53 photons

Static magnetic field (18 Gauss) lifts the $\sigma^+/\sigma$ degeneracy
Controlling the atom-cavity interaction time by selecting the atom’s velocity via Doppler effect sensitive optical pumping.

Rubidium level scheme with transitions implied in the selective depumping and repumping of one velocity class in the $F=3$ hyperfine state.

In green, velocity distribution before pumping, in red velocity distribution of atoms pumped in $F=3$, before they are excited in circular Rydberg state.
Detecting circular Rydberg states by selective field ionization

Adjusting the ionizing field allows us to discriminate two adjacent circular states. The global detection efficiency can be > 80%.
The ENS photon box (latest version)

In its latest version, the cavity has a damping time in the 100 millisecond range. Atoms cross it one at a time.

Cavity half-mounted...

...and fully-mounted
Orders of magnitude

\[ V \sim \pi w^2 L/4 \sim 4\lambda^3 \sim 700 \text{ mm}^3 \]

\[ \Omega \sim 10^{-6} \omega = 2\pi \times 50 \text{ kHz} \]

Parameter determined by geometric arguments

Number of vacuum Rabi flops during cavity damping time (best cavity):

\[ N_{RF} = \frac{\Omega T_C}{2\pi} \sim 5000 \]

Atom-cavity interaction time (depends on atom velocity):

\[ t_{int} \sim \frac{W}{v_{at}} \sim 10 \text{ to } 50 \mu s \]

Number of vacuum Rabi flops during atom-cavity interaction time:

\[ \Omega t_{int} / 2\pi \sim 1 \text{ to } 3 \]

Strong coupling in CQED

\[ \frac{1}{\Omega} < t_{int} \ll T_c \]

Many atoms cross one by one during \( T_c \)

\[ 3.1 \times 10^{-6} \text{ s} \quad 3.1 \times 10^{-5} \text{ s} \quad 10^{-1} \text{ s} \]
Cavity Quantum Electrodynamics: a stage to witness the interaction between light and matter at the most fundamental level.

One atom interacts with one (or a few) photon(s) in a box.

A sequence of atoms crosses the cavity, couples with its field and carries away information about the trapped light.

Photons bouncing on mirrors pass many many times on the atom: the cavity enhances tremendously the light-matter coupling.

The best mirrors in the world: more than one billion bounces and a folded journey of 40,000km (the earth circumference) for the light!

Photons are trapped for more than a tenth of a second!
I-D

Entanglement and quantum gate experiments in Cavity QED
Resonant Rabi flopping

\[
\cos\left(\frac{\Omega t}{2}\right)|e,0\rangle + \sin\left(\frac{\Omega t}{2}\right)|g,1\rangle
\]

Spontaneous emission and absorption involving atom-field entanglement

When \( n \) photons are present, the oscillation occurs faster (stimulated emission):

\[
\cos\left(\Omega \sqrt{n+1} \frac{t}{2}\right)|e,n\rangle + \sin\left(\Omega \sqrt{n+1} \frac{t}{2}\right)|g,n+1\rangle
\]

Simple dynamics of a two-level system (\(|e,n\rangle, |g,n+1\rangle\))
Rabi flopping in vacuum or in small coherent field: a direct test of field quantization

\[ P_e(t) = \sum_n p(n) \cos^2 \left( \frac{\Omega \sqrt{n+1} t}{2} \right) ; \quad p(n) = e^{-\bar{n}} \frac{\bar{n}^n}{n!} \]

\[ \bar{n} = 0 \quad (n_{th} = 0.06) \]
\[ \bar{n} = 0.40 \pm 0.02 \]
\[ \bar{n} = 0.85 \pm 0.04 \]
\[ \bar{n} = 1.77 \pm 0.15 \]

Useful Rabi pulses
(quantum « knitting »)

Initial state

$|e,0\rangle \rightarrow |e,0\rangle + |g,1\rangle$

$\pi/2$ pulse
creates atom-cavity entanglement

$|e,0\rangle \rightarrow \cos\left(\frac{\Omega t}{2}\right) |e,0\rangle + \sin\left(\frac{\Omega t}{2}\right) |g,1\rangle$

Brune et al, PRL 76, 1800 (96)

Microscopic entanglement
Useful Rabi pulses (quantum knitting)

\[ |e,0\rangle \rightarrow |g,1\rangle \]

\[ |g,1\rangle \rightarrow |e,0\rangle \]

\[ |g,0\rangle \rightarrow |g,0\rangle \]

\[ (|e\rangle + |g\rangle)|0\rangle \rightarrow |g\rangle (|1\rangle + |0\rangle) \]

\[ |e,0\rangle \rightarrow \cos(\frac{\Omega t}{2})|e,0\rangle + \sin(\frac{\Omega t}{2})|g,1\rangle \]

Brune et al, PRL 76, 1800 (96)
Useful Rabi pulses (quantum knitting)

\[ |e,0\rangle \rightarrow -|e,0\rangle \]
\[ |g,1\rangle \rightarrow -|g,1\rangle \]
\[ |g,0\rangle \rightarrow |g,0\rangle \]

2\pi pulse:
conditional dynamics and quantum gate

\[ |e,0\rangle \rightarrow \cos\left(\frac{\Omega t}{2}\right)|e,0\rangle + \sin\left(\frac{\Omega t}{2}\right)|g,1\rangle \]
Atom pair entangled by photon exchange

Electric field $F(t)$ used to tune atoms 1 and 2 in resonance with $C$ for times $t$ corresponding to $\pi/2$ or $\pi$ Rabi pulses

Hagley et al, P.R.L. 79,1 (1997)
The phase of the atomic fringes and their amplitude depend upon the state of field in C, which affect in different ways the probability amplitudes associated to states e and g.

Resonant classical $\pi/2$ pulses in auxiliary cavities $R_1$-$R_2$ (with adjustable phase offset $\phi$ between the two) prepare and analyse atom state superpositions.

Probabilities $P_e$ (or $P_g = 1 - P_e$) for finding atom in e or g oscillate versus $\phi$.
Effect of $2\pi$ Rabi flopping on Ramsey signal

*Cavity C* resonant with e-g (51-50) transition.

*Ramsey $R_1-R_2$ interferometer* resonant with g-i (50-49) transition.

2$\pi$ Rabi flopping on transition e-g in 1 photon field induces a $\pi$ phase shift between the g and i amplitudes.

$g-i$ fringes are inverted when photon number in C increases from 0 to 1.
Ramsey fringes conditioned to one photon in $C$

With proper phase choice, atom is detected in $g$ if $n = 0$, in $i$ if $n = 1$: quantum gate with photon (0/1) as control qubit and atom (i/g) as target qubit.

Experiment with 1st atom acting as source emitting 1 photon with probability 0.5 ($\pi/2$ pulse on $e$-$g$ transition) and 2nd probe atom undergoing Ramsey interference on g-i transition.
The quantum gate with photon as control bit realises a quantum non-demolition measurement (QND) of field.

Control bit (photon): $a = 0/1$

Target bit (atom): $b = 0$ (g) / 1 (i)

The atom carries away information about field energy without altering the photon number ($2\pi$ Rabi pulse). This is very different from usual photon detection, which is destructive.


Repetitive QND measurement of photons stored in super high Q cavity.
Second lecture (Wednesday)
Combining Rabi pulses for entanglement knitting

First atom prepares 1 photon with 50% probability (pulse $\pi/2$) and second atom reads photon by QND (pulse $2\pi$).

Third atom absorbs field (pulse $\pi$), producing a three atom correlation.