Anomalies, decoherence, quantum computation and black holes

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The Idea:

-- to show that qubit decoherence can in principle occur via quantum anomalies for certain systems. Such decoherence arises directly from quantum dynamics of the qubit system, not from interactions with environment.

--speculate on connection to black holes

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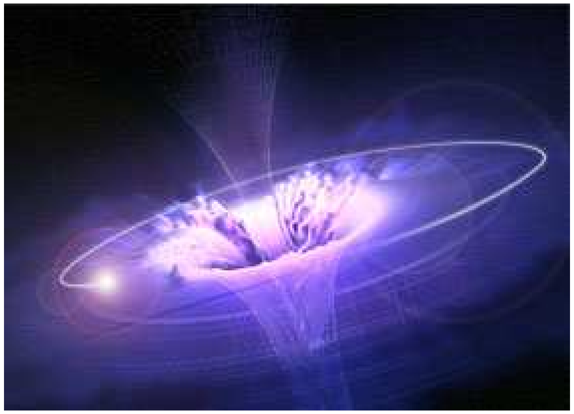
OUTLINE

1. Quantum Anomalies Primer

2. Qubit Decoherence from Anomalies
   (A toy model)

3. Connection to Black Holes?

4. Conclusions



1. Quantum Anomalies

- **Symmetries of classical systems sometimes cannot be preserved by quantization**
- **Classically conserved currents (charges) are not conserved at quantum level**
- **Gives rise to physically observable effects:**
  - **Axial Anomaly** $\pi \longrightarrow 2\gamma$ *(Adler, Bell, Jackiw '69)*
  - **U(1) problem in QCD** *('t Hooft '76)*
  - **Edge states in the quantum Hall effect**
  - **Hawking Radiation**
    - **Conformal Anomaly in 2-d**
    - **Gravitational anomaly** *(Robinson and Wilczek '05)*

1. Quantum Anomalies(cont’d)

- **Can be understood/calculated in several (equivalent) ways:**
  - **Perturbation theory** ***(Adler, Bell, Jackiw ’69)***
  - **Functional measure** ***(Fujikawa ‘79)***
  - **Non-self-adjointness of Hamiltonian** ***(Esteve ’02)***

1. Anomalies and Self-Adjointness Esteve '02

- Conserved Charge: $\dot{D} = \{D, H\} = 0 \qquad \dot{\hat{D}} = [\hat{D}, \hat{H}] = 0$
- In more detail:

$$\frac{d\langle\psi|\hat{D}\psi\rangle}{dt} = \langle\dot{\psi}|\hat{D}\psi\rangle + \langle\psi|\hat{D}\dot{\psi}\rangle$$
$$= i\langle\hat{H}\psi|\hat{D}\psi\rangle - i\langle\psi|\hat{D}\hat{H}\psi\rangle$$
$$= i\langle\psi|(\hat{H}^{+}\hat{D} - \hat{D}\hat{H})\psi\rangle$$
$$= i\langle\psi|[\hat{H},\hat{D}]\psi\rangle + i\langle\psi|(H^{+} - H)\hat{D}\psi\rangle$$

Anomaly: non-zero if *D* takes state outside domain of *H*

1.Anomalies in Quantum Mechanics

- Anomaly gives rise to non-conservation of *D* at quantum level
- Most common in field theory, but…
- Anomalies in quantum mechanics well studied in scale invariant quantum mechanics *(recently Camblong, Hernadez, Ordonez, ….)*:
  - $1/r^2$ potential
  - Delta f'n potential in 2-D
  - Scale invariant classically (coupling dimensionless)
  - Scale invariance not preserved in quantum theory

1. An Example

- $1/r^2$ potential with Hamiltonian

$$H\psi := \left(-\frac{\partial^2}{\partial r^2} - \frac{\lambda}{r^2} \right)\psi = E\psi$$

- Scale invariant: r→ar;E→E/a➔bound state spectrum continuous, unbounded below?
- Singularity requires care with boundary conditions
- Regularize, renormalize (eg cut-off L)
- Yields single, normalizable bound state:
- Existence of such a bound state breaks scale invariance

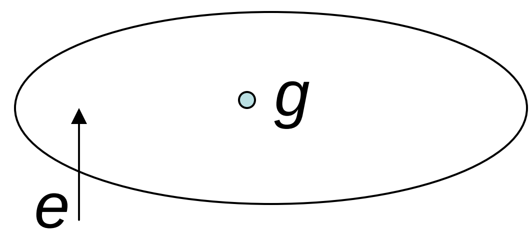
$$\psi_E(r) = \frac{2\beta}{\sqrt{\pi}}\sqrt{r}K_{1/2}(\beta r);$$

$$\beta = \sqrt{-2E} \qquad E \propto -\frac{\lambda}{L}$$

2.Qubit Decoherence from Anomalies

- Exotic qubit: electron bound in magnetic field produced by magnetic monopole

$$H = \frac{\left(\vec{p} - e\vec{A}\right)^2}{2m} - \frac{e}{2m}\vec{\sigma}\cdot\vec{B}$$



$$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{g}{r^2}\vec{e}_r$$

•***A*** is singularity free vector potential of the Dirac magnetic monopole➔*2eg* integer (quantization condition)

2. The Model (cont'd)

- Radial electron wave function of spin up component obeys:

$$H\psi := \left(-\frac{\nabla^2}{2m} - \frac{\lambda}{r^2}\right)\psi = E\psi$$

where $$\lambda := \frac{e^2 g^2}{2m}\left(1 + \frac{1}{|eg|}\right)$$

- Scale invariant quantum mechanics
- Singularity requires self-adjoint extension
- Regularize, renormalize (eg cut-off L)
- Yields single, normalizable bound state:

$$\psi_E(r) = \frac{2\beta}{\sqrt{\pi}}\sqrt{r}K_{1/2}(\beta r);$$

$$\beta = \sqrt{-2mE}$$

2. Qubit Decoherence (cont'd)

- Scale invariance clearly broken by existence of single bound state
- Anomaly: $\frac{dD}{dt} = \widehat{A} \qquad \widehat{A} = i(H^{+} - H)D$

$$\frac{d\langle \psi_E | D\psi_E \rangle}{dt} = \langle \psi_E | \widehat{A}\psi_E \rangle = \frac{e^2 g^2 \beta^2}{2m}\left(1 + \frac{1}{|eg|}\right)$$

- Dynamics of spin up bound state modified by anomaly:

$$|\psi_E(t)\rangle_{+} = e^{-i(E+iA)t}|\psi_E(0)\rangle_{+}$$

- Coefficient of spin down component stays zero
- Non-unitary evolution➔loss of probability (decoherence)
- Can do a careful analysis of qubit coupled to external oscillating magnetic field (spin flips)
- Non-unitarity signals incompleteness of model➔new physics

3.Black Hole Connection: Quantum Fields near Classical Horizon

- Consider generic BH metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -F(r)dt^2 + \frac{1}{F(r)} dr^2 + r^2 d\Omega^2$$

$$F(r_h) = 0; \qquad \kappa = F'(r_h)$$

- Klein-Gordon Equation for plane wave:

$$\varphi(r,t) = \psi(r) e^{i\omega t}; \qquad y \equiv r - r_h \to 0$$

$$-\frac{d^2\psi}{dy^2} - \frac{\lambda}{y^2}\psi = 0; \qquad \lambda \equiv \frac{\omega^2}{\kappa^2} + \frac{1}{4}$$

Camblong and Ordonez '05

3. Spherically Symmetric Vacuum Quantum Gravity

Berger et al. '72, Unruh '76; Gegenberg, G.K., Small '06

ADM Parametrization: $$ds^2 = X^2\left(-\sigma^2 dt^2 + \left(dx + N dt\right)^2\right)$$

Hamiltonian: $$H(t) = \int dx\left(\sigma\mathcal{G} + N\mathcal{F}\right) + H_B$$

Mass observable: $$\mathcal{M}(x) = \frac{l}{2G}\left(\left(\left(G\pi_X\right)^2 - \frac{\left(\varphi'\right)^2}{X^2}\right) + \frac{j(\varphi)}{l^2}\right);$$

Fix Spatial Diffeos: $$x := g(r)$$

3. Spherically Symmetric Vacuum Quantum Gravity

Hamiltonian Constraint: (Note: X^2 is conformal mode of metric)

$$\hat{\mathcal{M}}|\Psi\rangle \equiv \frac{1}{2G}\left(\hat{\pi}_X{}^2 - \frac{(\varphi')^2}{\hat{X}^2} + \frac{j}{l^2}\right)|\Psi\rangle = M_0|\Psi\rangle$$

- Independent quantum mechanical model at each spatial point
- Eigenvalue Equation of particle in $1/X^2$ potential
- Possible Representations:
  - Schrodinger Representation *(Louko and GK '06)*
  - Bohr Quantization *(Gegenberg, Small and GK '06)*

4.Conclusions

- **Quantum Anomalies can cause decoherence. Moral:**
  - **Avoid anomalous systems (magnetic monopoles) when designing quantum computers**
  - **Decoherence does not require interaction with external environment: anomaly rooted within dynamics of system itself**
- **Non-unitary evolution indicates description incomplete→need to find decay modes**
- **Decoherence via quantum anomaly may provide model for understanding information "loss" in black hole evaporation**